

FORMULATION OF THE BOUNDARY CONDITIONS AT THE WALL USING THE
LOCAL-SIMILARITY METHOD IN THE NUMERICAL MODELING OF THE FLOW
AND HEAT TRANSFER OF A VISCOUS INCOMPRESSIBLE LIQUID (LAMINAR
AND TURBULENT CONDITIONS)

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A new approach method of calculating the wall diffusional flows of the desired variables is proposed for the difference modeling of flow and heat transfer close to a solid surface at high Reynolds numbers.

Experience in the numerical solution of Navier-Stokes or Reynolds equations shows that one of the basic difficulties in the difference modeling of the flow and heat transfer of a viscous liquid at a solid surface is the lack of economical and yet sufficiently well-founded methods of formulating the boundary conditions at the wall in the case of high Reynolds numbers. The use of the traditional conditions of adhesion of the liquid to the wall is very problematic in this case if the limited (in terms both of memory and speed) possibilities of modern computers are taken into account. The wall-function method, which is widely used at present (see [1], for example), is based on the use of a logarithmic wall law and so is only suitable for describing fully developed turbulent near-wall flows, for which local energy equilibrium of the turbulent pulsations may be assumed. It is obvious that, in many cases, this condition is not satisfied. This may be illustrated by the flow of a cylinder in a transverse flow of viscous liquid. As is known, such flow is characterized by a clearly expressed influence of Re and the degree of turbulence of the incoming flow (external turbulence) on the position of the transition point from a laminar boundary layer developing at the cylinder surface to a turbulent state, and also on the point of flow breakaway from the cylinder surface. Note that external turbulence not only affects the position of the flow transition and breakaway points but also determines the processes occurring in the boundary layer itself. In particular, the properties of laminar and turbulent flow conditions appear equally close to the forward stagnation point of the cylinder.

Attempts to avoid the use of the wall-function method have been made in a number of works. Thus, the method developed in [2], which may conveniently be called the method of zero diffusion of the energy of turbulent pulsations (k) and its rate of dissipation (ϵ) at the wall, is based on the representation of k and ϵ in polynomial form: $k = A_1 y^2 + A_2 y^3 + O(y^4)$; $\epsilon = B_1 + B_2 y + O(y^2)$, where A_i and B_i are the coefficients of the polynomials; y is the distance from the normal to the wall. This method assumes that the gradients of k and ϵ at the wall are zero (the subscript w denotes the flow parameters at the wall). Note that the condition $(\partial k / \partial y)_w = 0$ is also adopted in the wall-function method; the condition $(\partial \epsilon / \partial y)_w \approx 0$, on the other hand, is more to be desired than actually found in practice. This primarily affects the accuracy of calculations of such important characteristics as the friction τ_w and heat flux q_w at the wall.

The accuracy with which τ_w and q_w are calculated may be improved by taking account of the near-wall diffusional transfer of the corresponding quantities in the viscous sublayer of the wall boundary layer, as proposed in [3] in developing the PSL procedure. The PSL procedure is based on the assumption that, even for a completely elliptical calculation region, there is a thin "parabolic sublayer" (hence the abbreviation PSL) in the immediate vicinity of the wall, and the change in static pressure in the flow across this sublayer is negligibly small. Thus, the pressure inside the parabolic sublayer is determined on the basis of solving the problem in the part of the calculation region external to the sublayer. Hence, the velocity component normal to the wall inside the sublayer may be found in this case not from the corresponding equation of variation in the momentum, as in the wall-function method and

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the method of zero diffusion of k and ε , but directly from the continuity equation. According to the data of [3], the thickness of the sublayer is chosen here so as to be larger than the entire low-Reynolds (in terms of the turbulent Reynolds number Re_T) region of flow. Note that, despite the simplicity of this approach, its realization in the difference solution of the initial system of Navier-Stokes or Reynolds equations is associated with definite difficulties, because in this case calculations must be performed on inscribed grids in the elliptical and parabolic (for the sublayer) regions, respectively.

To eliminate the deficiencies intrinsic to the above methods, in the present work a new method of formulating the boundary conditions at the wall is proposed for solving problems involving flow around a body of arbitrary form at high Re , in both laminar and turbulent conditions; essentially, the method is to use the local-similarity method in calculating the diffusional flows of the desired variables in the wall cells of the calculation region in the difference procedure. Since the local-similarity method was developed for an initial system of equations in the boundary-layer approximation (see [4, 5], for example), the use of this method is justified only in the part of the calculation region adjacent to the wall, where there is no flow breakaway, i.e., from the stagnation point of the flow to the point of flow breakaway from the solid surface.

The basic assumptions of the given method and the corresponding results are demonstrated preliminarily for the example of solving dynamic (calculating τ_w) and then also thermal (calculating q_w) problems in the case of laminar flow. Then the solution of the same problems is obtained for turbulent flow. The flow arriving at the body is assumed to be turbulent here; the flow from the stagnation point of the flow at the body surface to the transition point of the boundary layer to turbulent flow (according to the local Reynolds number Re_x) is assumed to be laminar, but with a specified influence from the turbulence of the external flow (this flow is conventionally called pseudolaminar); the flow beyond the transition point is defined as completely turbulent, so that the well-known methods of specifying the boundary conditions - for example, the wall-function method mentioned above - may be used on this section of the solid surface.

The boundary conditions at the wall formulated in this way allow the difference solution of the complete system of Navier-Stokes or Reynolds equations and energy equations to be obtained on the most general basis. In this analysis, the flow is assumed to be plane in both laminar and turbulent conditions, and the liquid is assumed to be incompressible with constant properties. The initial system of equations for the turbulent flow and heat transfer is closed by means of a two-parameter dissipative model of turbulence ($k - \varepsilon$), although other models of turbulence of differential type may be used here, in principle.

Laminar Conditions

First, the dynamic problem is solved. Using a coordinate system fixed in the body, the initial equations of motion are written, in the boundary-layer approximation, in the form of the Folkner-Scan equation

$$f''' + ff'' + \beta(1 - f'^2) = 0, \quad (1)$$

where $f'(\eta) = u/u_e$; $\eta = u_e y / \sqrt{2\nu s}$; $s = \int u_e dx$; $\beta = (2s/u_e) du_e/ds$; the subscript e denotes the external boundary of the boundary layer at the solid surface ($\eta \rightarrow \infty$); the notation $()'$ denotes the first derivative with respect to η (the number of primes denotes the order of the derivative).

The solution of Eq. (1) is constructed with the boundary conditions

$$\eta = 0, \quad f(0) = f'(0) = 0; \quad \eta \rightarrow \infty, \quad f'(\infty) \rightarrow 1 \quad (2)$$

for some specified value of the Folkner-Scan parameter β , which, although it depends on the law of variation of the velocity component tangential to the solid surface $u_e(x)$, is assumed to be constant for $x = \text{const}$. The constraint on the extent of the calculation region is determined by the parameter $\beta = -0.1988$, corresponding to the point of flow breakaway from the solid surface. In the stagnation point at the solid surface (let $x = 0$), the parameter $\beta = 1$. Then, when $-0.1988 < \beta \leq 1$, the solution of Eq. (1) with the boundary conditions in Eq. (2) allows the dimensional value of the friction at the wall to be found

$$\tau_w^* = \mu (\partial u / \partial y)_w = (\mu u_e^2 / \sqrt{2\nu s}) f''(0). \quad (3)$$

Under the assumption that the pressure in the calculation region of variation in β is determined from the solution of the initial system of equations in the Navier-Stokes form,

the dependence $\beta(x)$ is found. By definition, $\beta = (2s/u_e)du_e/ds$. Using the velocity of the unperturbed flow U_∞ (here and below, the subscript ∞ corresponds to parameters of the unperturbed flow) and the size of the body in the flow L as the characteristic quantities, so that $Re = U_\infty L/\nu$, the expression for β is rewritten in the form

$$\beta = \left(\int_0^x u_e dx \right) (2/u_e^2) du_e/dx,$$

and hence, taking into account that the Bernoulli equation along the streamline coinciding with the solid surface gives

$$u_e = [2(p_{0e} - p_e)]^{1/2}, \quad (4)$$

where p_e is the static pressure and p_{0e} is the stagnation pressure (at $x = 0$), it is found that

$$\beta = -\frac{dp_e}{dx} (p_{0e} - p_e)^{-3/2} \int_0^x (p_{0e} - p_e)^{1/2} dx. \quad (5)$$

Note that, in principle, it is possible to determine u_e from the solution of the Navier-Stokes equation; however, the numerical value of u_e in this case depends on the size of the wall cell employed. The use of Eq. (4) seems preferable, since the pressure variation in the direction normal to the wall is extremely small at considerable distance from the wall, and hence depends weakly on the size of the calculation-region wall cells chosen in solving the Navier-Stokes equations.

The solution procedure reduces to determining the distribution of friction over the solid surface of $f''(0)$ at fixed β , within the limits of the wall-cell step in the direction along the solid surface. The solution of Eq. (1) with the boundary conditions in Eq. (2) for $f''(0; \beta)$ is not difficult. Values of $f''(0; \beta)$ have been given in numerous works (see [4], for example). In the calculations, the boundary value of the coordinate η_e at which $u/u_e = 0.9999$ ($\eta \rightarrow \infty$) is also found. It does not exceed $\eta_e = 6.2$ when $\beta = -0.1988$. From Eq. (3), introducing the dimensionless friction $\tau_w = \tau_w^*/(\rho U_\infty^2)$, where ρ is the density of the liquid, and taking account of Eq. (5), the following result is obtained

$$\tau_w = 2^{1/4} (p_{0e} - p_e)^{3/4} f''(0) / [Re \int_0^x (p_{0e} - p_e)^{1/2} dx]^{1/2}. \quad (6)$$

The friction at the wall determined in this way is then used to calculate the diffusional flow of the velocity component u in the wall cells of the difference grid in constructing the algorithm for solving the initial system of Navier-Stokes equations over the whole calculation region. Note that the constraint on the size y_1 of the difference-grid cell closest to the wall follows from determining the self-similar coordinate η ; taking account of Eq. (4), under the assumption that $p_e = p_1$; $p_{0e} = p_{01}$, it may be written in the form

$$y_1 \approx \eta_e \left[\int_0^x [2(p_{01} - p_1)]^{1/2} dx / [Re (p_{01} - p_1)] \right]^{1/2}, \quad (7)$$

where η_e corresponds to the theoretical boundary-layer thickness $\eta \rightarrow \infty$.

Now turning to the formulation of the boundary conditions for the temperature, the dimensionless temperature $T = (T^* - T_w^*) / (T_\infty^* - T_w^*)$ is introduced, where an asterisk denotes a dimensional quantity; then, at the wall, $T_w = 0$. The derivative of the temperature taken along the normal to the wall is in this case the dimensionless heat flux q_w to the wall in the form of the Nusselt number $Nu_w = \alpha L / \lambda = (\partial T / \partial y)_w$. For more precise determination of Nu_w and hence of the diffusional temperature flux at the wall, the local-similarity method must subsequently be used in solving the total energy equation in the calculation region, as in solving the dynamic problem. In the boundary-layer approximation, the energy equation takes the form

$$\Theta'' + Pr f \Theta' = 0. \quad (8)$$

In Eq. (8), the function Θ is introduced in the same way as the dimensionless temperature T . The difference between Θ and T is that Θ , like the function f , depends on the self-similar coordinate $\eta = u_e y / \sqrt{2\nu s}$, where $s = \int u_e dx$. The dependence $f(\eta)$ is found from the solution of Eq. (1) with the boundary conditions in Eq. (2). The order of Eq. (8) may be reduced, to give

$$\Theta'(\eta) = C \exp[-F(\eta)], \quad (9)$$

where $F(\eta) = \text{Pr} \int_0^\eta f(\eta) d\eta$; C is a constant.

The solution of Eq. (9) is sought with the boundary conditions

$$\eta = 0, \quad \Theta(0) = 0; \quad \eta \rightarrow \infty, \quad \Theta(\infty) \rightarrow 1 \quad (10)$$

(the temperature is assumed to be constant at the external boundary of the boundary layer).

Using Eq. (10), the constant C in Eq. (9) may be determined. Finally, Eq. (9) takes the form

$$\Theta'(\eta) = \left[\int_0^\infty \exp[-F(\eta)] d\eta \right]^{-1} \exp[-F(\eta)], \quad (11)$$

and hence it follows that when $\eta = 0$

$$C = \Theta'(0) = 1 / \int_0^\infty \exp \left[-\text{Pr} \int_0^\eta f(\eta) d\eta \right] d\eta.$$

The heat flux to the wall is now determined. In dimensionless form, it is written in terms of $\Theta'(0)$ as the Nusselt number $\text{Nu}_w = (u_e L / \sqrt{2\nu s}) \Theta'(0)$. Taking account of the expression for β in the Folkner-Scan equation, the Nusselt number may be brought to the form $\text{Nu}_w = [L \Theta'(0) / \sqrt{\beta \nu}] \sqrt{du_e/dx}$, where u_e is determined from Eq. (4) and β from Eq. (5). Then the final relation obtained for Nu_w is

$$\text{Nu}_w / \sqrt{\text{Re}} = \Theta'(0) (p_{0e} - p_e)^{1/2} \left[2 \int_0^x (p_{0e} - p_e)^{1/2} dx \right]^{-1/2}, \quad (12)$$

and hence it follows that, at the stagnation point of the flow, when $x = 0$ ($\beta = 1$), there is a singularity. Writing u_e in the vicinity of the stagnation point as in the form $u_e = Bx$, where B is the velocity gradient at the stagnation point, the result obtained for Nu_w at this point is

$$\text{Nu}_w / \sqrt{\text{Re}_B} = \Theta'(0), \quad (13)$$

where $\text{Re}_B = BL^2/\nu$ and B is found from the numerical solution of the system of Navier-Stokes equations in the calculation region.

The derivative $\Theta'(0)$ appearing in Eqs. (12) and (13) is determined from the solution of Eq. (11) with the boundary conditions in Eq. (10). For the stagnation point, with $\beta = 1$, $\Theta'(0)$ depends on the Prandtl number Pr. The results of calculating $\Theta'(0; \text{Pr})$ for this point are given in [4], for example. The results of calculating $\Theta'(0; \beta; \text{Pr})$ for $-0.1988 < \beta \leq 1$ are also known [5].

Thus, determining the heat flux to the body surface by the local-similarity method reduces to finding $\Theta'(0)$ as a function of β and Pr. Since the tables for $\Theta'(0; \beta; \text{Pr})$ are fairly unwieldy, the use of the following procedure is proposed.

1. As a result of solving the dynamic problem, $\beta(x)$ and $f''(0)$ are found for each wall cell of the calculation region; at the stagnation point, $f''(0) = 1.2326$, and $\Theta'(0)$ may be approximated by the formula: $\Theta'(0) = 0.57\text{Pr}^{0.4}$.

2. Beyond the stagnation point, for the parameter β specified in each wall cell, the Cauchy problem for Eq. (1) is solved with the boundary conditions $f(0) = f'(0) = 0$; $f''(0)$ is specified as a function of β and determined by interpolation in accordance with the calculational data presented in [4].

3. Plotting the profile $f(\eta)$ for the specified β , $\Theta'(0)$ is found from Eq. (11).

4. The heat flux to the wall is determined from Eq. (12) or Eq. (13) and then used to calculate the temperature field in solving the complete energy equation in the calculation region.

Turbulent Conditions

Suppose that the flow arriving at the body is turbulent. The turbulence of this flow is taken into account when using the local-similarity method by including additional terms in

the system of equations of motion and energy equations in the boundary-layer approximation, as compared with Eqs. (1) and (8); these terms characterize the transfer of turbulence from the external flow to the wall. In the coordinate system fixed in the body (the same notation as for laminar flow is used here), the equation of change in momentum in combination with the continuity equation is written in the form of a modified Folkner-Scan equation

$$\partial/\partial\eta[(1 + \nu_T/\nu)f'''] + ff'' + \beta(1 - f'^2) = 0. \quad (14)$$

As noted above, if the turbulent viscosity ν_T appearing in Eq. (14) is determined on the basis of the assumptions made in the dissipative model of turbulence, i.e., assuming that $\nu_T = C_\mu k^2/\varepsilon$, it may be approximately assumed that $k \sim y^2$ and $\varepsilon = \varepsilon_w \sim y$ in the viscous sub-layer of the wall boundary layer. This means that $\nu_T \sim y^4$ when $y \rightarrow 0$ and $\nu_T \sim y^3$ with increase in ε on moving away from the wall so that $\varepsilon \gg \varepsilon_w$. At the same time, $\partial k/\partial y \approx 0$ and $\varepsilon \sim 1/y$ in the completely developed turbulent (logarithmic) layer of the given flow. Hence $\nu_T \sim y$ here. Since, it is difficult to predict in advance how the external flow turbulence will influence the structure of the given flow between the stagnation point and the transition point from laminar to developed turbulent conditions (that is, in the region of pseudolaminar flow as defined above), the simplest phenomenological model of turbulence for this type of flow will be used in establishing the dependence $\nu_T(y)$ in the pseudolaminar section [6]; in this model, the turbulent viscosity changes linearly with distance from the wall, that is

$$\nu_T/\nu = \gamma \left[(4 \operatorname{Re}/3) (k_e/u_e^2) \int_0^x u_e dx \right]^{1/2} \eta, \quad (15)$$

where γ is a constant of the model.

It is interesting to compare Eq. (15) with the formula for $\nu_T = C_\mu k^2/\varepsilon$. Using the relations for k and ε specified by the wall-function method for developed turbulent flow in a logarithmic layer [1], it is found that

$$\nu_T/\nu = \kappa C_\mu^{1/4} \left[2 \operatorname{Re} (k_e/u_e^2) \int_0^x u_e dx \right]^{1/2} \eta \quad (16)$$

or

$$\nu_T/\nu = [C_\mu^{3/4} \sigma_\varepsilon (C_{\varepsilon 2} - C_{\varepsilon 1})/\kappa] \left[2 \operatorname{Re} (k_e/u_e^2) \int_0^x u_e dx \right]^{1/2} \eta. \quad (17)$$

Comparing Eq. (15) with Eqs. (16) and (17), the relation between the constant of the phenomenological model γ and the constants of the dissipative model may be established: $\gamma = (3/2)^{1/2} \kappa C_\mu^{1/4} = (3/2)^{1/2} C_\mu^{3/4} \sigma_\varepsilon (C_{\varepsilon 2} - C_{\varepsilon 1})/\kappa$. Hence, when $\kappa = 0.4$, $C_\mu = 0.09$, $C_{\varepsilon 2} = 1.92$, $C_{\varepsilon 1} = 1.44$, the constant γ is 0.268 or $0.24\sigma_\varepsilon$. With variation in σ_ε from the minimum value $\sigma_\varepsilon = 1$ to the usual value $\sigma_\varepsilon = 1.3$ taken in the dissipative model, it follows that $\gamma = 0.24$ and 0.31 , respectively. Note that, according to experimental data for flow in the vicinity of the stagnation point of the flow at the surface of a blunt body, $\gamma = 0.17-0.2$ [6].

As follows from Eq. (15), ν_T/ν is a function of η only when

$$(2k_e/u_e^2) \int_0^x u_e dx = \beta_T = \text{const} \quad (18)$$

or taking account of Eq. (4)

$$\beta_T = k_e (\rho_{0e} - \rho_e)^{-1} \int_0^x [2(\rho_{0e} - \rho_e)]^{1/2} dx.$$

It is evident from Eq. (18) that there is an indeterminacy in calculating β_T at the stagnation point ($\beta = 1$). Resolving this by means of the representation $u_e = Bx$, as in the case of laminar flow, β_T is determined at the stagnation point as $\beta_T = k_e/B$.

With dependences $\beta(x)$ and $\beta_T(x)$ specified from the solution of the system of Reynolds equations, the solution of Eq. (14) is found with the boundary conditions in Eq. (2). If Eq. (15) is written as $\nu_T/\nu = A\eta$, where $A = \gamma(2\beta_T \operatorname{Re}/3)^{1/2}$, Eq. (14) takes the form

$$(1 + A\eta)f''' + (A + f)f'' + \beta(1 - f'^2) = 0. \quad (19)$$

The procedure for solving Eq. (19) with the boundary conditions in Eq. (2) reduces to calculating the function $f(\eta)$ and its derivatives in searching for $f''(0)$, so that $\varphi[f''(0)] =$

$|1 - f'(\infty)| < \delta$, where δ is the permissible error in the satisfaction of the boundary condition $f'(\infty) \rightarrow 1$. The friction at the wall is determined from Eq. (6) on the basis of the result obtained for $f''(0)$, and is then used in calculating the diffusional flow of the velocity component u in the wall cells of the calculation grid, analogously as for laminar flow. The coordinate y_1 of the calculation-grid cell closest to the wall is then determined from Eq. (7). Assuming that the diffusion of k at the wall is zero, it may be supposed that $k_1 \approx k_e$ ($\eta \rightarrow \infty$). Then, using the dependence for ν_T adopted in the dissipative model of turbulence and Eq. (15), the dissipation rate ϵ_1 at the point closest to the wall is determined

$$\epsilon_1 = (3/2)^{1/2} C_\mu k^3 / (\gamma y_1). \quad (20)$$

Note that Eq. (20) is identical to the dependence for ϵ used in describing the state of local isotropy of the dissipating eddies.

To solve the thermal problem, the energy equation is used, in the form (in the boundary-layer approximation)

$$[1/\text{Pr} + (1/\text{Pr}_T) \nu_T/\nu] \Theta'' + [f' + (1/\text{Pr}_T) \partial/\partial \eta (\nu_T/\nu)] \Theta' = 0. \quad (21)$$

Taking account of Eq. (15), Eq. (21) takes the form

$$\Theta'(\eta) = C_T \exp[-F_T(\eta)], \quad (22)$$

where

$$F_T(\eta) = \text{Pr} \int_0^\eta (f' + A/\text{Pr}) [1 + (\text{Pr}/\text{Pr}_T) A \eta]^{-1} d\eta;$$

$$C_T = \Theta'(0) = 1 / \left[\int_0^\infty \exp\left(-\int_0^\eta \text{Pr} F_T(\eta) d\eta\right) d\eta \right].$$

From the result for $\Theta'(0)$, the heat flux to the wall is found in Nusselt-number form from Eq. (12) or Eq. (13). Overall, the procedure for solving the thermal problem in the case where the influence of external turbulence on the heat transfer is taken into account is not different from the analogous solution procedure for laminar flow conditions. Note that the results of calculating the thermal flow at the stagnation point at the surface of a blunt body in the form $\text{Nu}_w / \sqrt{\text{Re}_B}$ ($\text{Re}_B = BL^2/\nu$, B is the velocity gradient at the stagnation point) are given in [6] for $\text{Pr} = 0.73, 1, \text{Pr}_T = 1$, and $A = \gamma(2\beta_T \text{Re}/3)^{1/2}$ varying in the range $0.01 \leq A < 100$. From the result obtained for Nu_w at the wall, the diffusional temperature flow at the wall is determined by solving the Reynolds and energy equations over the whole calculation region.

NOTATION

x, y , coordinates along the normal to the solid surface; η , self-similar coordinate; u , velocity component in the direction x , $u = u_e f'(\eta)$; U , characteristic velocity; k , kinetic energy of turbulent pulsations; ϵ , energy dissipation rate of turbulent pulsations; $T(x, y)$, $\theta(\eta)$, temperature; p , pressure; ρ , density; L , characteristic dimension of body; τ , friction; q , heat flux; β , Folkner-Scan parameter; β_T , parameter determining the turbulent intensity in the wall layer; B , velocity gradient at the stagnation point; μ , dynamic viscosity; ν , kinematic viscosity; α , heat-transfer coefficient; λ , thermal conductivity; ν_T , turbulent viscosity; κ , Karman constant; γ, A , constants of the phenomenological model of turbulence: $C_\mu, \sigma_\epsilon, C_{\epsilon_1}, C_{\epsilon_2}$, constants of the dissipative model of turbulence; $\text{Re}, \text{Re}_T, \text{Re}_x, \text{Re}_B$, Reynolds numbers: $\text{Re} = U_\infty L/\nu$, $\text{Re}_T = k^2/(\nu\epsilon)$, $\text{Re}_x = u_e x/\nu$, $\text{Re}_B = BL^2/\nu$; Pr , Prandtl number; Pr_T , turbulent Prandtl number; Nu , Nusselt number. Indices: w , wall; ∞ , unperturbed flow; e , external boundary of boundary layer; 0 , flow stagnation; 1 , first wall point of the difference grid; $()'$, derivative with respect to η ; $*$, dimensional value.

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